Neutral Higgs Boson Pair-Production and Trilinear Self-Couplings in the MSSM at ILC and CLIC Energies

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Abstract

We study pair-production as well as the triple self-couplings of the neutral Higgs bosons of the Minimal Supersymmetric Standard Model (MSSM) at the Future International Linear e^+e^- Collider (ILC) and Compact Linear Collider (CLIC). The analysis is based on the reactions $e^+e^- \to b\bar{b}h_ih_i, t\bar{t}h_ih_i$ with $h_i = h, H, A$. We evaluate the total cross-section for both $b\bar{b}h_ih_i$, $t\bar{t}h_ih_i$ and calculate the total number of events considering the complete set of Feynman diagrams at tree-level. We vary the triple couplings $\kappa\lambda_{hhh}$, $\kappa\lambda_{Hhh}$, $\kappa\lambda_{hAA}$, $\kappa\lambda_{HAA}$, $\kappa\lambda_{hHH}$ and $\kappa\lambda_{HHH}$ within the range $\kappa = -1$ and $+2$. The numerical computation is done for the energies expected at the ILC with a center-of-mass energy 500, 1000, 1600 GeV and a luminosity 1000 fb^{-1}. The channels $e^+e^- \to b\bar{b}h_ih_i$ and $e^+e^- \to t\bar{t}h_ih_i$ are also discussed to a center-of-mass energy of 3 TeV and luminosities of 1000 fb^{-1} and 5000 fb^{-1}.

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I. INTRODUCTION

The search for Higgs bosons is one of the principal objectives of present and future high-energy colliders, such as the International Linear Collider (ILC) [1, 2, 3, 4], Compact Linear Collider (CLIC) [5] and Large Hadron Collider (LHC) [6, 7]. It has been demonstrated in Ref. [8] that physics at the LHC and e^+e^- ILC will be complementary to each other in many respects. In many cases, the ILC can significantly improve the LHC measurements.

The Higgs boson [9, 10, 11, 12, 13] plays an important role in the Standard Model (SM) [14, 15, 16] because it is responsible for generating the masses of all the elementary particles (leptons, quarks, and gauge bosons). However, the Higgs-boson sector is the least tested in the SM, in particular the Higgs boson self-interaction. If Higgs bosons are responsible for breaking the symmetry from $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$, it is natural to expect that other Higgs bosons are also involved in breaking other symmetries. One of the more attractive extensions of the SM is Supersymmetry (SUSY) [17, 18, 19], mainly because of its capacity to solve the naturalness and hierarchy problems while maintaining the Higgs bosons elementary.

The theoretical framework of this paper is the Minimal Supersymmetric extension of the Standard Model (MSSM), which doubles the spectrum of particles of the SM, and the new free parameters obey simple relations. The scalar sector of the MSSM [20, 21] requires two Higgs doublets, thus the remaining scalar spectrum contains the following physical states: two CP-even Higgs scalares (h^0, H^0) with $M_h \leq M_H$, one CP-odd Higgs scalar (A^0) and a charged Higgs pair $(H^\pm)$. The Higgs sector is specified at tree-level by fixing two parameters which can be chosen as the mass of the pseudoscalar M_A and the ratio of vacuum expectation values of the two doublets $\tan\beta = v_2/v_1$; then, the masses M_h, M_H and $M_{H^\pm}$ and the mixing angle of the neutral Higgs sector α can be fixed. However, since radiative corrections produce substantial effects on the predictions of the model [22, 23, 24, 25, 26, 27, 28, 29], it is necessary to also specify the squark masses, which are assumed to be degenerated.

In particular, all the triple self-couplings of the physical Higgs particles can be predicted (at the tree level) in terms of M_A and $\tan\beta$. Once a light Higgs boson is discovered, the measurement of these triple couplings can be used to reconstruct the Higgs potential of the MSSM.

The triple Higgs self-couplings can be measured directly in pair-production of Higgs particles at hadron and high energy e^+e^- colliders. In proton collisions at the LHC [7],

Higgs pairs can be produced through double Higgs-strahlung off W and Z bosons, WW and ZZ fusion, and gluon-gluon fusion. The triple Higgs boson couplings are involved in a large number of processes at e^+e^- collider [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42]:

$$\begin{aligned} \text{double Higgs-strahlung} &: e^+e^- \to ZH_iH_j \quad \text{and} \quad ZAA \quad [H_{i,j} = h, H], \\ \text{triple Higgs production} &: e^+e^- \to AH_iH_j \quad \text{and} \quad AAA, \\ WW \text{ fusion} &: e^+e^- \to \bar{\nu}_e\nu_e H_iH_j \quad \text{and} \quad \bar{\nu}_e\nu_e AA, \end{aligned} \tag{1}$$

these three-body processes have been evaluated extensively. However, the inclusion of four-body processes with heavy fermions b and t ($e^+e^- \to b\bar{b}h_ih_i, t\bar{t}h_ih_i, h_i = h, H, A$) is important in order to know its impact on three-body channels and also to search for new relations that may have a clear signature of the Higgs boson production. In the studied processes, the MSSM Higgs bosons are radiated by $b(\bar{b})$ and $t(\bar{t})$ quarks at future e^+e^- colliders [43, 44, 45, 46, 47, 48] with a c.m. energy in the range of 500 to 1600 GeV, as in the case of the ILC [1] and of 3 TeV to the CLIC machine [5].

In this paper, we study the pair-production as well as the triple self-couplings of the Higgs bosons of the Minimal Supersymmetric Standard Model (MSSM) at the Future International Linear e^+e^- Collider (ILC) [1] and Compact Linear Collider (CLIC) [5]. The analysis is based on the reactions $e^+e^- \to b\bar{b}h_ih_i, t\bar{t}h_ih_i$ with $h_i = h, H, A$. We evaluate the total cross-section for both $b\bar{b}h_ih_i$, $t\bar{t}h_ih_i$ and calculate the total number of events considering the complete set of Feynman diagrams at tree-level. We vary the triple couplings $\kappa\lambda_{hhh}$, $\kappa\lambda_{Hhh}$, $\kappa\lambda_{hAA}$, $\kappa\lambda_{HAA}$, $\kappa\lambda_{hHH}$ and $\kappa\lambda_{HHH}$ within the range $\kappa = -1$ and $+2$. The numerical computation is done for the energies expected to be available at the ILC with a center-of-mass energy 500, 1000, 1600 GeV and a luminosity 1000 fb^{-1}. Our analysis is also extended to a center-of-mass energy 3 TeV and luminosities of 1000 fb^{-1} and 5000 fb^{-1}. We consider $\tan\beta = 35$ and $M_A = 400$ GeV.

The Higgs couplings with quarks, the largest couplings in the MSSM, are directly accessible in the processes where the Higgs boson is radiated off bottom quarks, $e^+e^- \to b\bar{b}hh, b\bar{b}HH, b\bar{b}AA$ as well as in the processes $e^+e^- \to t\bar{t}hh, t\bar{t}HH, t\bar{t}AA$ where the Higgs boson is radiated off top quarks. These processes depend on the Higgs boson triple self-couplings, which could lead us to obtain the first non-trivial information on the Higgs potential. We are interested in finding regions that could allow the observation of the

$b\bar{b}hh, b\bar{b}HH, b\bar{b}AA$ and $t\bar{t}hh, t\bar{t}HH, t\bar{t}AA$ processes at future linear e^+e^- colliders energies: ILC and CLIC. We consider the complete set of Feynman diagrams at tree-level (Figs. 1-3) and use the CALCHEP [49] packages to evaluate the amplitudes and cross-sections of the processes $e^+e^- \to b\bar{b}h_ih_i, t\bar{t}h_ih_i$.

This paper is organized as follows: In Sec. II, the Higgs boson interaction Lagrangian and the self-couplings are presented. In Sec. III, we study the triple Higgs boson self-coupling through the processes $e^+e^- \to b\bar{b}hh(HH, AA)$ and $e^+e^- \to t\bar{t}hh(HH, AA)$ at future e^+e^- colliders energies (ILC/CLIC) and, finally, we summarize our results in Sec. IV.

II. HIGGS BOSON INTERACTION LAGRANGIAN

The Higgs sector of MSSM includes five physical fields: two neutral CP-even Higgs scalar (h^0, H^0), one neutral CP-odd Higgs scalar (A^0) and a charged Higgs pair $(H^\pm)$. The Higgs boson interaction lagrangian has the form

$$\mathcal{L}^{Higgs}_{Int} = \mathcal{L}^{(3)}_{Int} + \mathcal{L}^{(4)}_{Int}, \tag{2}$$

where $\mathcal{L}^{(3)}_{Int}$ is the lagrangian of the triple Higgs boson interactions, while $\mathcal{L}^{(4)}_{Int}$ is the lagrangian of the quartic Higgs boson interactions. Explicitly, $\mathcal{L}^{(3)}_{Int}$ is:

$$\begin{aligned}\mathcal{L}^{(3)}_{Int} &= \frac{\lambda_{hhh}}{3!}hhh + \frac{\lambda_{hhH}}{2!}hhH + \frac{\lambda_{hHH}}{2!}hHH + \frac{\lambda_{HHH}}{3!}HHH + \frac{\lambda_{hAA}}{2!}hAA \\ &+ \frac{\lambda_{HAA}}{2!}HAA + \lambda_{hH^+H^-}hH^+H^- + \lambda_{HH^+H^-}HH^+H^-. \end{aligned}\tag{3}$$

The triple Higgs self-coupling of the MSSM can be parameterized as [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42],

$$\begin{aligned}\lambda_{hhh} &= 3\cos 2\alpha \sin(\beta+\alpha) + 3\frac{\epsilon}{M_Z^2}\frac{\cos\alpha}{\sin\beta}\cos^2\alpha, \\ \lambda_{Hhh} &= 2\sin 2\alpha \sin(\beta+\alpha) - \cos 2\alpha\cos(\beta+\alpha) + 3\frac{\epsilon}{M_Z^2}\frac{\sin\alpha}{\sin\beta}\cos^2\alpha, \\ \lambda_{HHh} &= -2\sin 2\alpha\cos(\beta+\alpha) - \cos 2\alpha\sin(\beta+\alpha) + 3\frac{\epsilon}{M_Z^2}\frac{\cos\alpha}{\sin\beta}\sin^2\alpha, \\ \lambda_{HHH} &= 3\cos 2\alpha\cos(\beta+\alpha) + 3\frac{\epsilon}{M_Z^2}\frac{\sin\alpha}{\sin\beta}\sin^2\alpha, \end{aligned}\tag{4}$$

$$
\begin{aligned}
\lambda_{hAA} &= \cos 2\beta \sin(\beta+\alpha) + \frac{\epsilon}{M_Z^2}\frac{\cos\alpha}{\sin\beta}\cos^2\beta, \\
\lambda_{HAA} &= -\cos 2\beta \cos(\beta+\alpha) + \frac{\epsilon}{M_Z^2}\frac{\sin\alpha}{\sin\beta}\cos^2\beta,
\end{aligned}
$$

where $\epsilon \approx \frac{3G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \log \frac{M_S^2}{m_t^2}$ and the connection between the mixing angles α and β is given by

$$
\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2 + \epsilon/\cos\beta} \quad \text{with} \quad -\frac{\pi}{2} \leq \alpha \leq 0, \tag{5}
$$

as a function of M_A and $\tan\beta$.

III. HIGGS BOSONS PAIR PRODUCTION WITH MODIFIED TRIPLE SELF-COUPLING

In this section we present numerical results for $e^+e^- \to b\bar{b}hh(HH, AA)$ and $e^+e^- \to t\bar{t}hh(HH, AA)$ with double Higgs bosons production. We carry out the calculations using the framework of the Minimal Supersymmetric Standard Model at the future linear e^+e^- colliders. We use the CALCHEP [49] packages for calculations of the matrix elements and cross-sections. These packages provide automatic computation of the cross-sections and distributions in the MSSM as well as their extensions at tree-level. We consider the high energy stage of a possible Future Linear e^+e^- Collider (ILC) with $\sqrt{s} = 500, 1000, 1600$ GeV and design luminosity 1000 fb^{-1}. For the numerical computation, we have adopted the following parameters: the angle of Weinber $\sin^2\theta_W = 0.232$, the mass ($m_b = 4.5\ GeV$) of the bottom quark, the mass ($m_t = 175\ GeV$) of the top quark, the mass ($m_{Z^0} = 91.2\ GeV$) of the Z^0, the mass ($M_A = 200 - 400\ GeV$) of the CP-odd Higgs scalar and $\tan\beta = 35$.

A. Triple Higgs Bosons Self-Coupling Via $e^+e^- \to b\bar{b}hh(HH, AA)$

To illustrate our results on the sensitivity to the $hhh, Hhh, HHh, HHH, hAA, HAA$ triple modified Higgs bosons self-coupling, we show the κ dependence of the total cross-section for $e^+e^- \to b\bar{b}hh(HH, AA)$ in Fig. 4. We consider one representative value of the Higgs boson mass, $M_A = 400\ GeV$, and $\tan\beta = 35$ with a center-of-mass energy of $\sqrt{s} = 500, 1000, 1600$ GeV and vary the triple couplings $\kappa\lambda_{hhh}, \kappa\lambda_{Hhh}, \kappa\lambda_{HHh}, \kappa\lambda_{HHH}, \kappa\lambda_{hAA}, \kappa\lambda_{HAA}$ within the range $\kappa = -1$ and $+2$. In all cases,

the cross-section is sensitive to the value of the triple couplings, as well as in the case of the process $e^+e^- \to b\bar{b}hh$, where for large values of M_A (the decoupling limit), the corresponding MSSM triple coupling approaches the SM triple coupling [50]. However, the cross-section and its sensibility to $\lambda_{hhh}, \lambda_{hHH}$ decrease with increasing collider energy for $e^+e^- \to b\bar{b}hh$, while for $e^+e^- \to t\bar{t}HH(AA)$, the sensitivity to $\lambda_{hHH}, \lambda_{HHH}$ and $\lambda_{hAA}, \lambda_{HAA}$ increases with rising collider energy. In particular, we can see in Fig. 5 for $e^+e^- \to b\bar{b}hh(HH, AA)$, the maximum cross-section (for $M_A = 400$ GeV and $\tan\beta = 35$) is reached for $\sqrt{s} \sim 500$ GeV and 1800 GeV respectively, which is consistent with Ref. [50] in the case where h is identified as the Higgs boson of the standard model. As an indicator of the order of magnitude, in Tables I-III we present the Higgs boson number of events (we have to multiplicate by the corresponding Branching Ratios to obtain the observable number of events) for several Higgs boson masses M_A, center-of-mass energy and κ values and for a luminosity of $1000 fb^{-1}$. If we consider the $h \to b\bar{b}$ decay for $M_h < 130$ GeV, we have some opportunity to detect this process. In this region, the number of events is small but sufficient to detect $e^+e^- \to b\bar{b}hh \to b\bar{b}b\bar{b}b\bar{b}$. The $BR(h \to b\bar{b}) \sim 0.6$ and the background for 6 b-jet are small.

Finally, for completeness, in Fig. 6 we include a contour plot for the number of events of the studied processes as a function of $\sqrt{s}$ and κ with $M_A = 400$ GeV and $\tan\beta = 35$. These contour are obtained from Tables I-III. Because in major parts of the MSSM parameter space the neutral Higgs bosons H and A are quite heavy, it is difficult to detect the processes $e^+e^- \to b\bar{b}HH(AA)$ when the relevant mechanism is $e^+e^- \to b\bar{b}hh$, as shown in the number of events given in Tables I-III.

B. Triple Higgs Bosons Self-Coupling Via $e^+e^- \to t\bar{t}hh(HH, AA)$

As in the case of the processes $e^+e^- \to b\bar{b}hh(HH, AA)$, Fig. 7 shows the variation of the cross-sections for $e^+e^- \to t\bar{t}hh(HH, AA)$ with $\kappa\lambda_{hhh}, \kappa\lambda_{Hhh}, \kappa\lambda_{HHh}, \kappa\lambda_{HHH}, \kappa\lambda_{hAA}, \kappa\lambda_{HAA}$, $\kappa = -1$ to $+2$, $500 \leq \sqrt{s} \leq 1600$ GeV, $M_A = 400$ GeV and $\tan\beta = 35$. The cross-section and its sensibility to the triple self-coupling on the energy range considered is dependent on the behavior of the cross-section with the center-of-mass energy determined by the position of the maximum. In Fig. 8, we can see that the maximum cross-section is reached for $\sqrt{s} \approx 1100$ GeV in the case of $e^+e^- \to t\bar{t}hh$, while for $e^+e^- \to t\bar{t}HH(AA)$ it is reached for

TABLE I: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 0.5$.

Total Production of Higgs Boson Pairs	$e^+e^- \to b\bar{b}hh(HH, AA)$		$\kappa = 0.5$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	26 (-,-)	17 (7,7)	10 (7,7)
250	26 (-,-)	17 (7,7)	10 (7,7)
300	26 (-,-)	17 (7,7)	10 (7,7)
350	26 (-,-)	17 (7,7)	10 (7,7)
400	26 (-,-)	17 (7,7)	10 (7,7)

TABLE II: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 1(MSSM)$.

Total Production of Higgs Boson Pairs	$e^+e^- \to b\bar{b}hh(HH, AA)$		$\kappa = 1(MSSM)$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	33 (-,-)	19 (7,7)	11 (7,7)
250	33 (-,-)	19 (7,7)	11 (7,7)
300	33 (-,-)	19 (7,7)	11 (7,7)
350	33 (-,-)	19 (7,7)	11 (7,7)
400	33 (-,-)	19 (7,7)	11 (7,7)

$\sqrt{s} \approx 1800\ GeV$. For the process $e^+e^- \to t\bar{t}hh$ (see Fig. 8, Ref. [50]) and for large values of M_A (the decoupling limit), the corresponding MSSM triple coupling approaches the SM triple coupling [50]. On the other hand, for $t\bar{t}HH(AA)$, the cross-section of the order 10^{-5} fb is small and is difficult to measure in the collider. In Tables IV-VI to indicate the order of magnitude, we present the Higgs boson number of events (we have to multiplicate for the corresponding Branching Ratios to obtain the observable number of events) for several Higgs

TABLE III: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 1.5$.

Total Production of Higgs Boson Pairs	$e^+e^- \to b\bar{b}hh(HH, AA)$		$\kappa = 1.5$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	42 (-,-)	22 (7,7)	12 (7,7)
250	42 (-,-)	22 (7,7)	12 (7,7)
300	42 (-,-)	22 (7,7)	12 (7,7)
350	42 (-,-)	22 (7,7)	12 (7,7)
400	42 (-,-)	22 (7,7)	12 (7,7)

boson masses M_A, center-of-mass energy and κ values and for a luminosity of $1000 fb^{-1}$.

For $e^+e^- \to t\bar{t}hh$, the most favorable situation is for a center-of-mass energy of 1100 GeV and $M_h < 130\ GeV$, but in this case, the Branching Ratios for the four decay modes make this process very much suppressed.

Finally, we include a contour plot for the number of events of the studied processes in the $(\sqrt{s}, \kappa)$ plane with $M_A = 400\ GeV$ and $\tan\beta = 35$ in Fig. 9. These contours are obtained from Tables IV-VI. Since in major parts of the MSSM parameter space the neutral Higgs bosons H and A are quite heavy, it is difficult to detect the processes $e^+e^- \to t\bar{t}HH(AA)$ when the relevant mechanism is $e^+e^- \to t\bar{t}hh$ for $\sqrt{s} \approx 1100\ GeV$ as shown in Tables IV-VI.

C. Triple Higgs Boson Self-Coupling Via $e^+e^- \to b\bar{b}hh(HH, AA), t\bar{t}hh(HH, AA)$ at CLIC energies

In this subsection we analyze the triple Higgs self-coupling $\kappa\lambda_{hhh}, \kappa\lambda_{Hhh}, \kappa\lambda_{HHh}, \kappa\lambda_{HHH}, \kappa\lambda_{hAA}, \kappa\lambda_{HAA}$ via the processes $e^+e^- \to b\bar{b}hh(HH, AA)$ and $e^+e^- \to t\bar{t}hh(HH, AA)$ for energies expected at the CLIC [5]. Figures 10 and 11 show the total cross-section for the double Higgs-strahlung in e^+e^- collisions, $e^+e^- \to b\bar{b}hh(HH, AA)$ and $e^+e^- \to t\bar{t}hh$ as a function of κ for the c.m. energy of $\sqrt{s} = 3\ TeV$, $M_A = 400\ GeV$ and $\tan\beta = 35$. From these figures, we observe that the production cross-section of both

TABLE IV: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 0.5$.

Total Production of Higgs Boson Pairs	$e^+e^- \to t\bar{t}hh(HH, AA)$		$\kappa = 0.5$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	- (-,-)	21 (-,-)	17 (-,-)
250	- (-,-)	21 (-,-)	17 (-,-)
300	- (-,-)	21 (-,-)	17 (-,-)
350	- (-,-)	21 (-,-)	17 (-,-)
400	- (-,-)	21 (-,-)	17 (-,-)

TABLE V: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 1(MSSM)$.

Total Production of Higgs Boson Pairs	$e^+e^- \to t\bar{t}hh(HH, AA)$		$\kappa = 1(MSSM)$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	- (-,-)	23 (-,-)	19 (-,-)
250	- (-,-)	23 (-,-)	19 (-,-)
300	- (-,-)	23 (-,-)	19 (-,-)
350	- (-,-)	23 (-,-)	19 (-,-)
400	- (-,-)	23 (-,-)	19 (-,-)

processes is small, as it is of the order of $10^{-3}\ fb$.

Finally, in Tables VII and VIII we present the Higgs boson (hh, HH, AA) number of events for several κ values, luminosities of 1000 and 5000 fb^{-1} and center-of-mass energy $\sqrt{s} = 3\ TeV$ (we have to multiplicate by the corresponding Branching Ratios to obtain the observable number of events). It is evident from Figures 10 and 11 and Table VII that it would be difficult to obtain a clear signal of the processes $e^+e^- \to b\bar{b}hh(HH, AA)$,

TABLE VI: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $\mathcal{L} = 1000\ fb^{-1}$ and $\kappa = 1.5$.

Total Production of Higgs Boson Pairs	$e^+e^- \to t\bar{t}hh(HH, AA)$		$\kappa = 1.5$
$M_A(GeV)$	$\sqrt{s} =$ 500 GeV	$\sqrt{s} =$ 1000 GeV	$\sqrt{s} =$ 1600 GeV
200	- (-,-)	26 (-,-)	21 (-,-)
250	- (-,-)	26 (-,-)	21 (-,-)
300	- (-,-)	26 (-,-)	21 (-,-)
350	- (-,-)	26 (-,-)	21 (-,-)
400	- (-,-)	26 (-,-)	21 (-,-)

TABLE VII: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $M_A = 400\ GeV$, $\sqrt{s} = 3\ TeV$ and $\mathcal{L} = 1000\ fb^{-1}$.

Total Production of Higgs Boson Pairs	$M_A = 400\ GeV$,		$\sqrt{s} = 3\ TeV$	
κ	$e^+e^- \to b\bar{b}hh$	$e^+e^- \to b\bar{b}HH$	$e^+e^- \to b\bar{b}AA$	$e^+e^- \to t\bar{t}hh$
0.5	5	3	3	7
1	5	3	3	8
1.5	5	3	3	9

$e^+e^- \to t\bar{t}hh(HH, AA)$ at energies of a future linear collider such as CLIC, after having considered the background, except for $\sqrt{s} = 3\ TeV$ and very high luminosity ($\mathcal{L} = 5000\ fb^{-1}$), as shown in Table VIII. However, for the CLIC center-of-mass energy, the WW double Higgs fusion process [7, 30, 31, 32, 33, 34], which increases with rising $\sqrt{s}$, can be exploited by large energies and luminosities and would be the preferred mechanism to measure the triple Higgs self-couplings.

TABLE VIII: Total production of Higgs boson pairs in the MSSM for $\tan\beta = 35$, $M_A = 400\ GeV$, $\sqrt{s} = 3\ TeV$ and $\mathcal{L} = 5000\ fb^{-1}$.

Total Production of Higgs Boson Pairs	$M_A = 400\ GeV$,		$\sqrt{s} = 3\ TeV$	
κ	$e^+e^- \to b\bar{b}hh$	$e^+e^- \to b\bar{b}HH$	$e^+e^- \to b\bar{b}AA$	$e^+e^- \to t\bar{t}hh$
0.5	25	15	14	35
1	26	16	15	41
1.5	28	18	17	47

IV. CONCLUSIONS

e^+e^- linear colliders represent a possible opportunity for the triple Higgs boson self-coupling analysis. Therefore, we have analyzed the neutral Higgs bosons self-couplings with a complete set of the tree-level Feynman diagrams in the framework of the MSSM. The dependence of the triple Higgs bosons self-coupling λ_{hhh}, λ_{Hhh}, λ_{hAA}, λ_{HAA}, λ_{hHH} and λ_{HHH} on $\tan\beta$ and energy $\sqrt{s}$ were evaluated.

The extended Higgs spectrum in supersymmetric theories gives rise to a plethora of triple couplings. The hhh coupling is generally quite different from the one expected in the standard model. It can be measured in Higgs double production at Future International Linear e^+e^- Colliders (ILC). Even though the e^+e^- cross sections are below the hadronic cross sections, the strongly reduced number of background events renders the search easier for the Higgs-pair signal, through $b\bar{b}b\bar{b}$ final states for instance, in the e^+e^- environment than in jetty LHC final states. For sufficiently high luminosities the first phase of these colliders with an energy of 500 GeV will allow the experimental analysis of self-couplings for Higgs bosons in the intermediate mass range. Other couplings between heavy and light MSSM Higgs bosons can be measured as well, though only in restricted areas of the $(\sqrt{s}, \kappa)$ plane as illustrated in the sets of Figs. 6 and 9.

In summary, we have analyzed the triple Higgs bosons self-coupling at future e^+e^- colliders energies, with the reactions $e^+e^- \to b\bar{b}hh(HH, AA)$ and $e^+e^- \to t\bar{t}hh(HH, AA)$. The first is the most important after considering the h decay, and is statistically sufficient for an

accurate determination of κ. In the case of $e^+e^- \to t\bar{t}hh(HH, AA)$, after we consider the t and h decays the final number of events is small; although they produce a non-vanishing number of events, statistically it is insufficient for the determination of κ. In these conditions the number of events is small but our results have never been reported in the literature before and could be of relevance for the scientific community.

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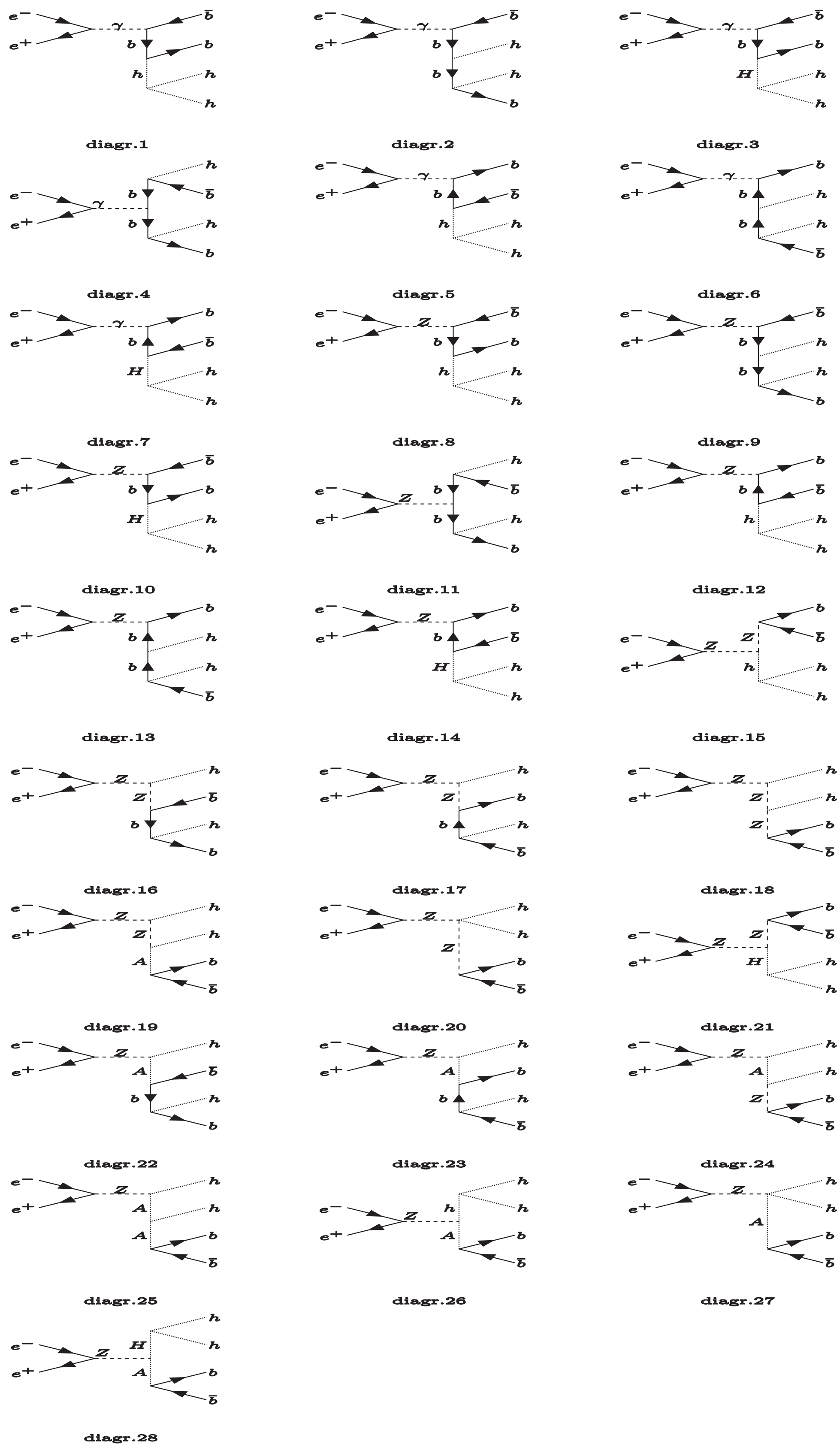

FIG. 1: Feynman diagrams at tree-level for $e^+e^- \to b\bar{b}hh$. The diagrams for $e^+e^- \to t\bar{t}hh$ are similar.

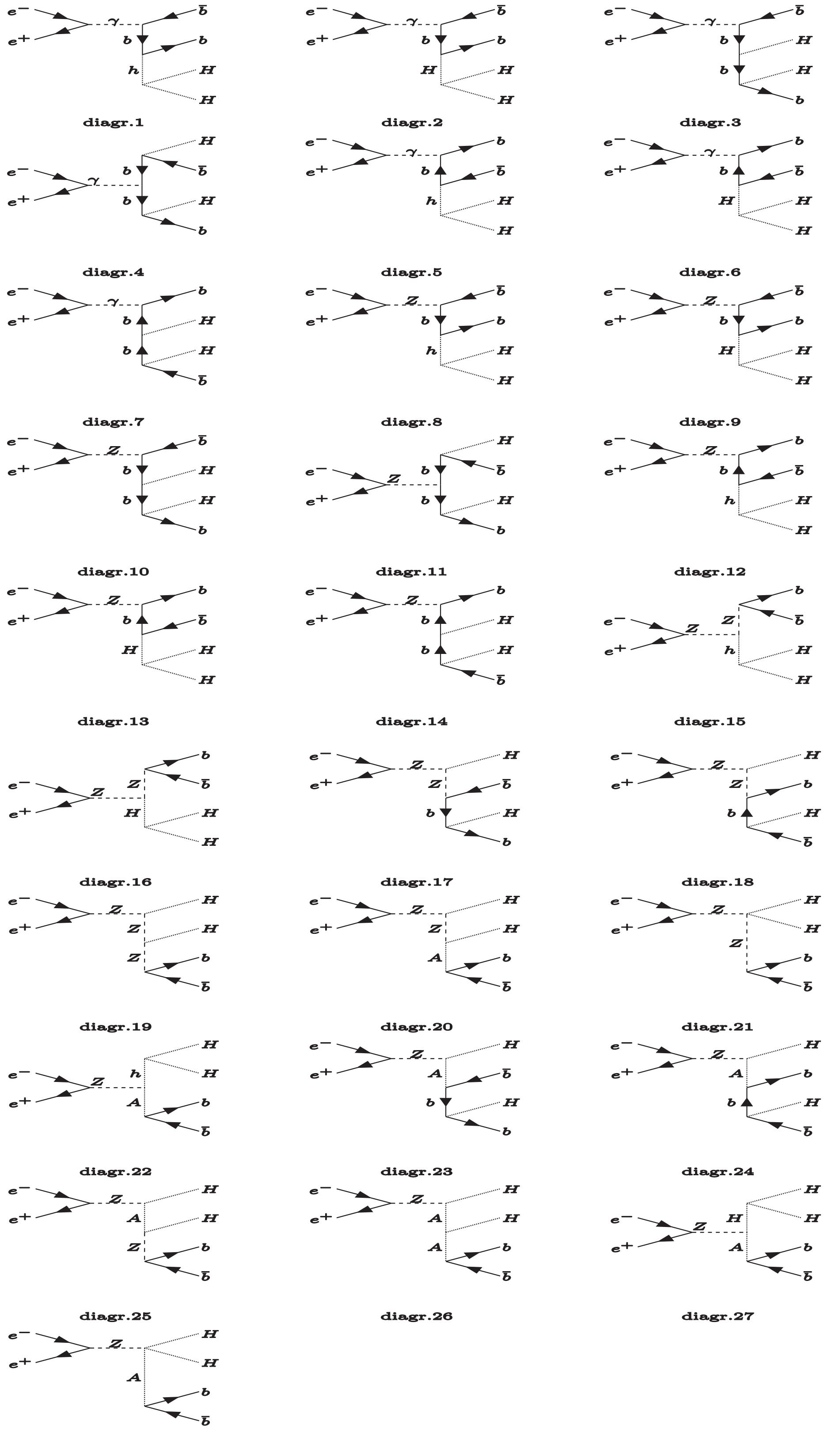

FIG. 2: Feynman diagrams at tree-level for $e^+e^- \to b\bar{b}HH$. The diagrams for $e^+e^- \to t\bar{t}HH$ are similar.

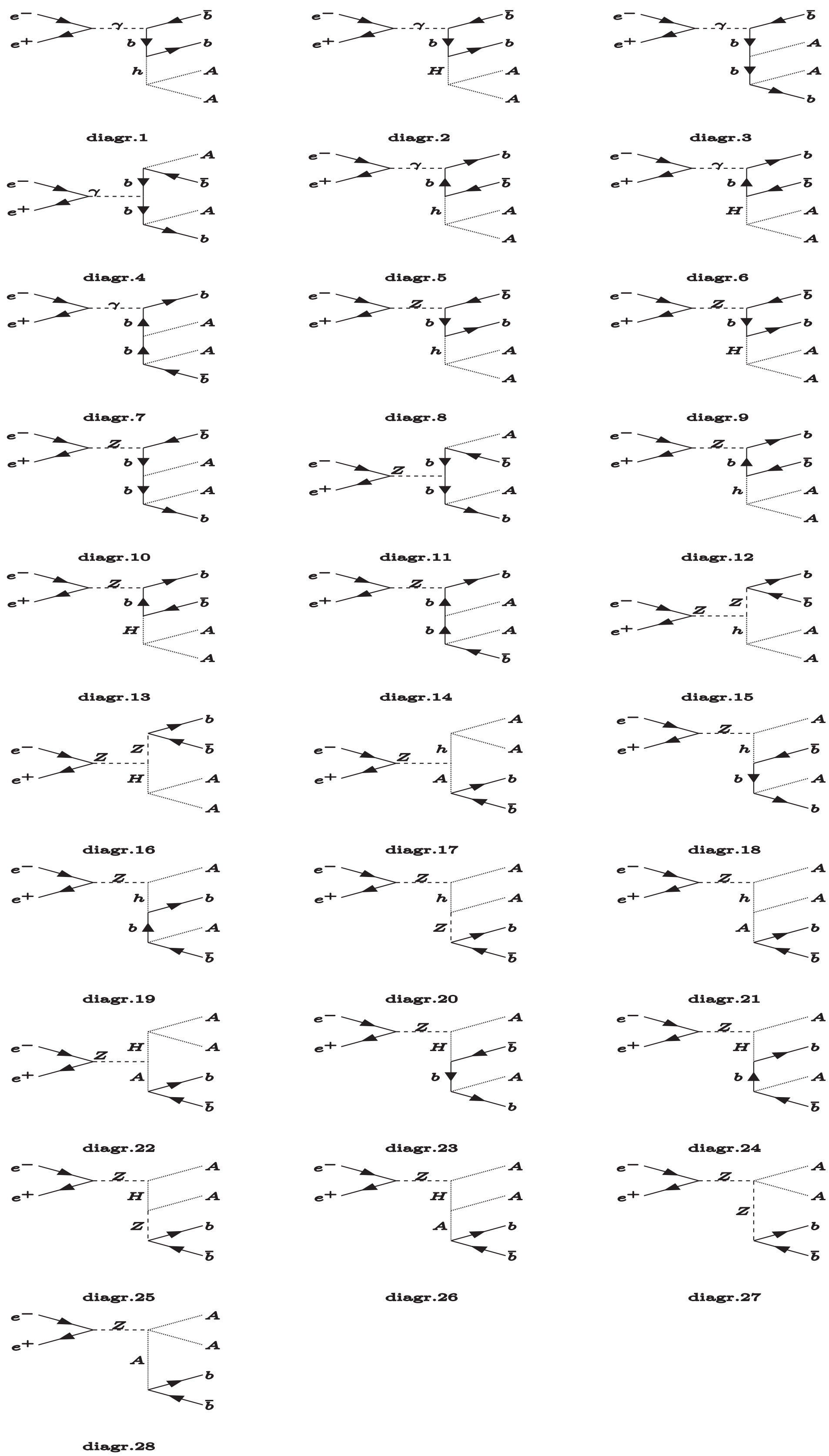

FIG. 3: Feynman diagrams at tree-level for $e^+e^- \to b\bar{b}AA$. The diagrams for $e^+e^- \to t\bar{t}AA$ are similar.

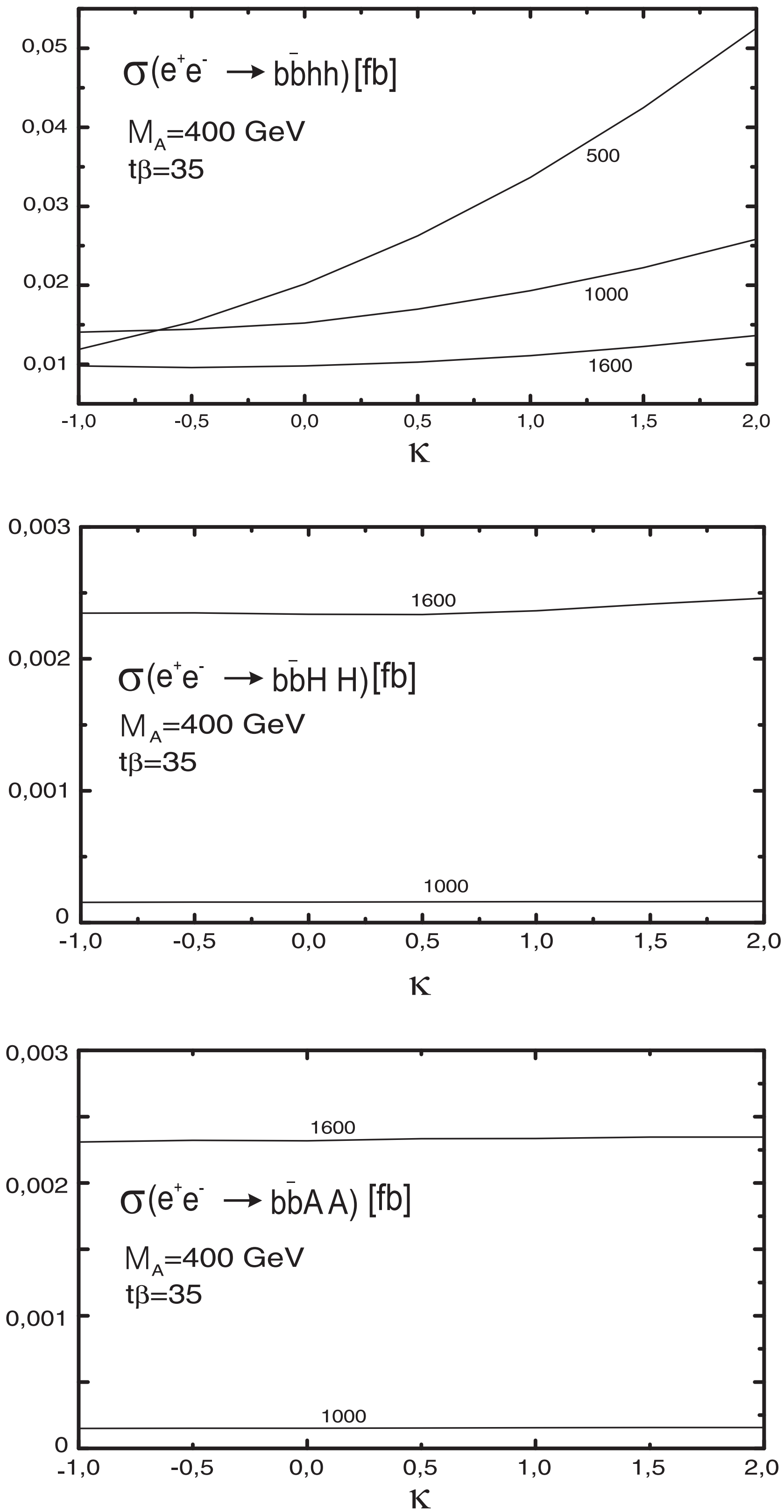

FIG. 4: Variation of the cross-section $\sigma(b\bar{b}hh, b\bar{b}HH, b\bar{b}AA)$ with the modified triple coupling $\kappa\lambda_{hhh}$, $\kappa\lambda_{Hhh}$ at a collider energy of $\sqrt{s} = 500, 1000, 1600\ GeV$ with $M_A = 400\ GeV$ and $\tan\beta = 35$.

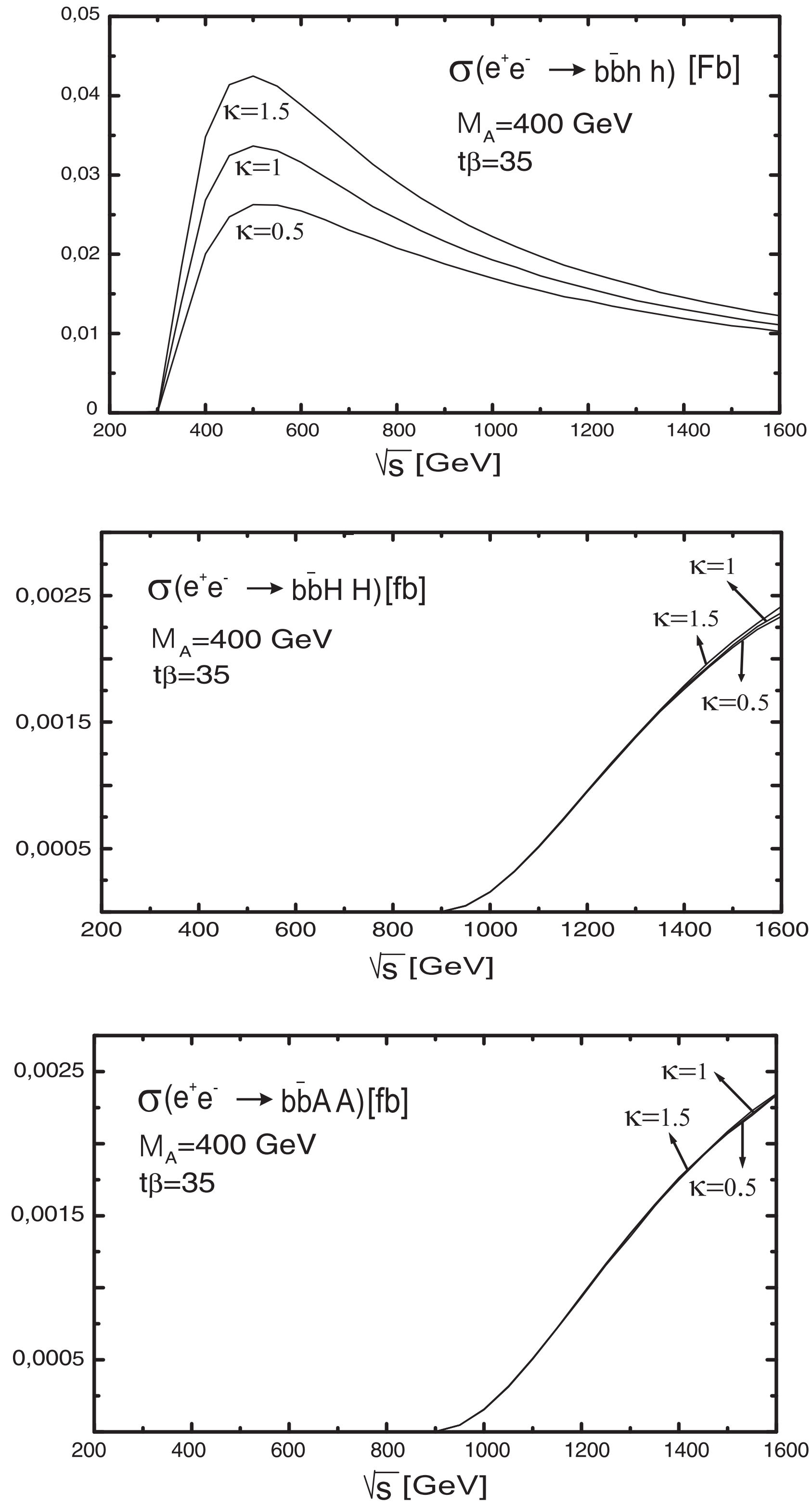

FIG. 5: The dependence of the cross-section on center-of-mass energy $\sqrt{s}$ for $M_A = 400\ GeV$ and $\tan\beta = 35$. The variation of the cross-section for modified triple couplings $\kappa\lambda_{hhh}$, $\kappa\lambda_{Hhh}$ is indicated by $\kappa = 0.5, 1.5$.

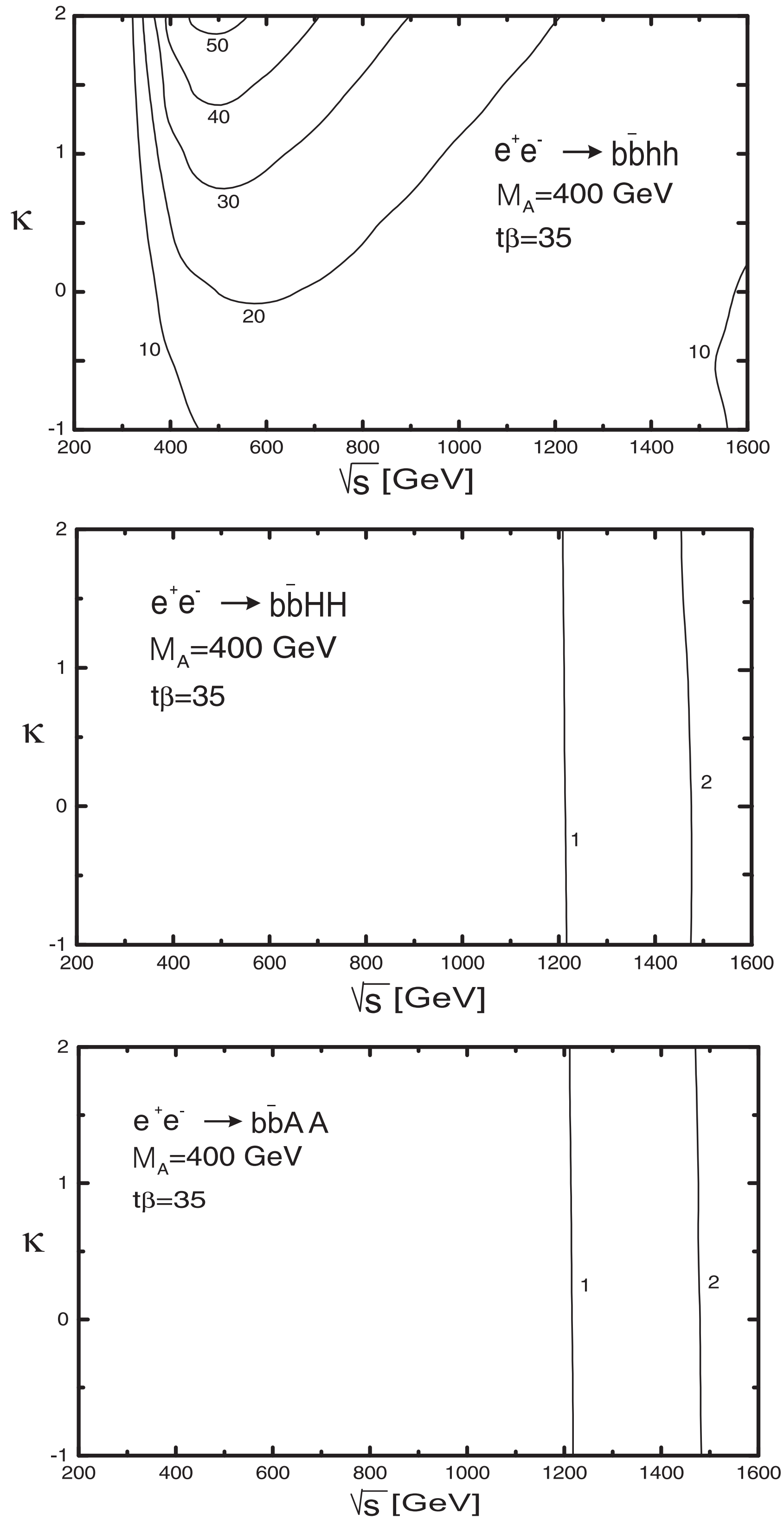

FIG. 6: Contour plot for the number of events of the processes $e^+e^- \to b\bar{b}hh, b\bar{b}HH, b\bar{b}AA$ as a function of $\sqrt{s}$ and κ.

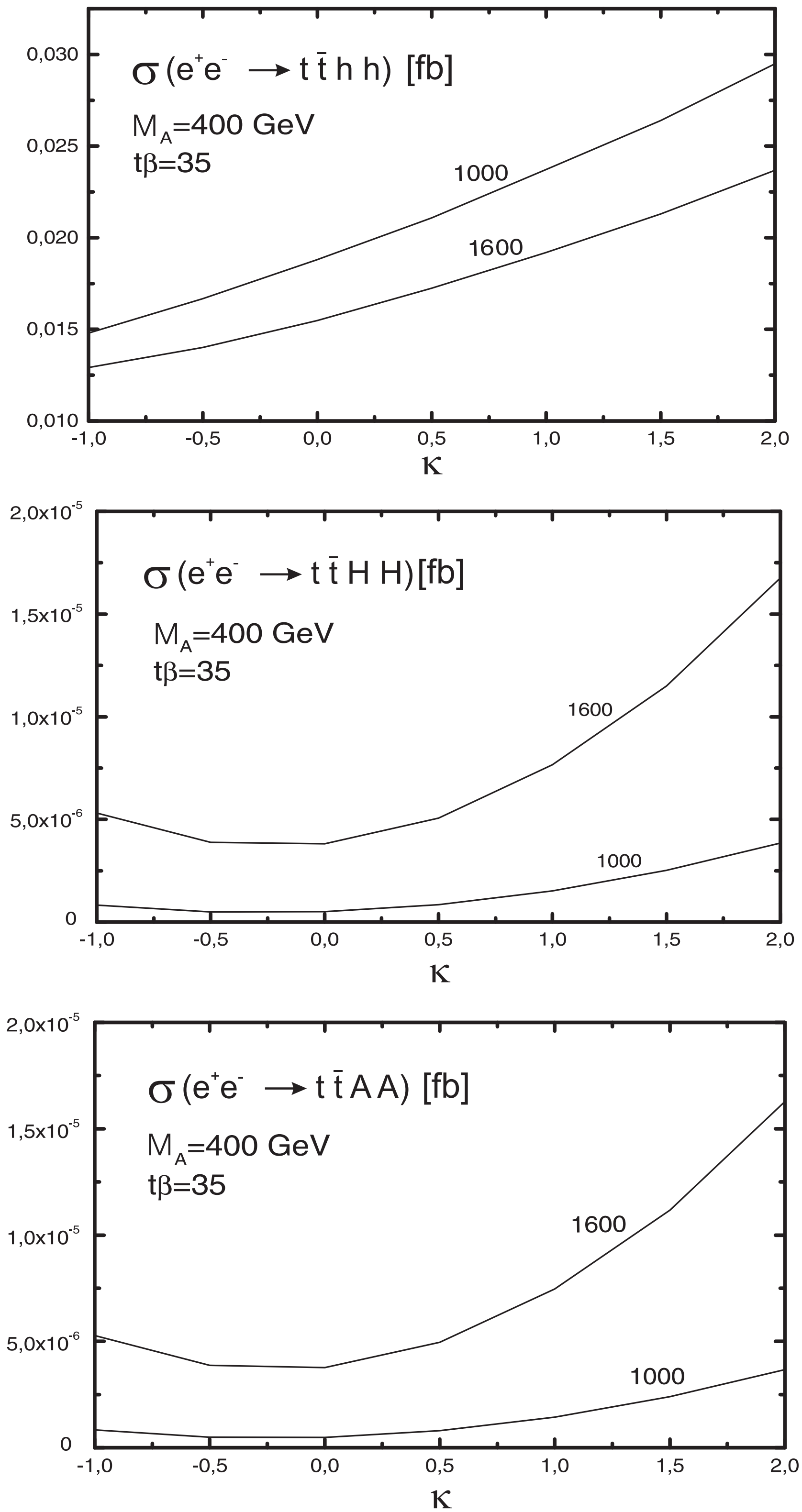

FIG. 7: The same as in Fig. 4, but for the processes $e^+e^- \to t\bar{t}hh, t\bar{t}HH, t\bar{t}AA$.

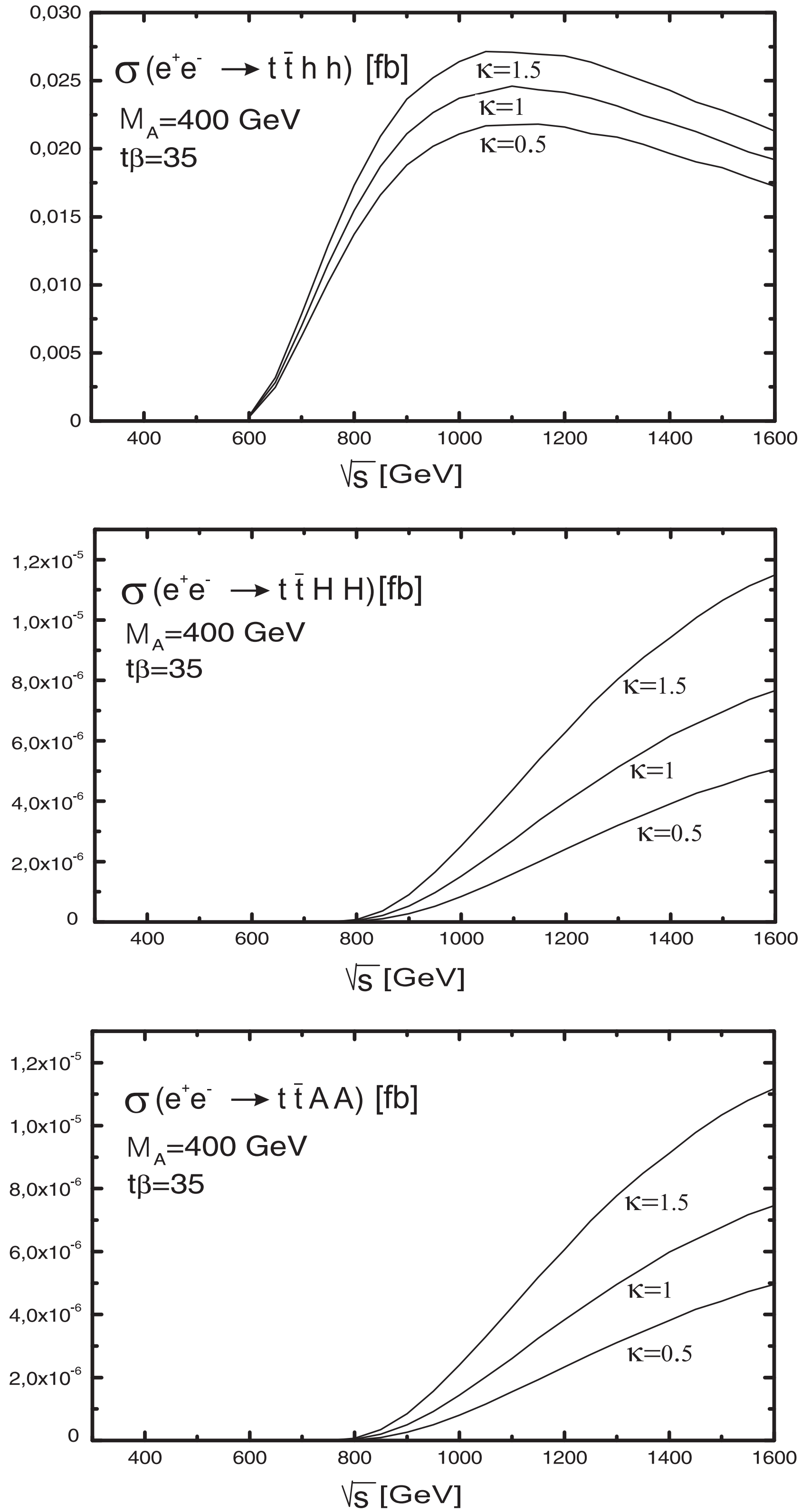

FIG. 8: The same as in Fig. 5, but for the processes $e^+e^- \to t\bar{t}hh, t\bar{t}HH, t\bar{t}AA$.

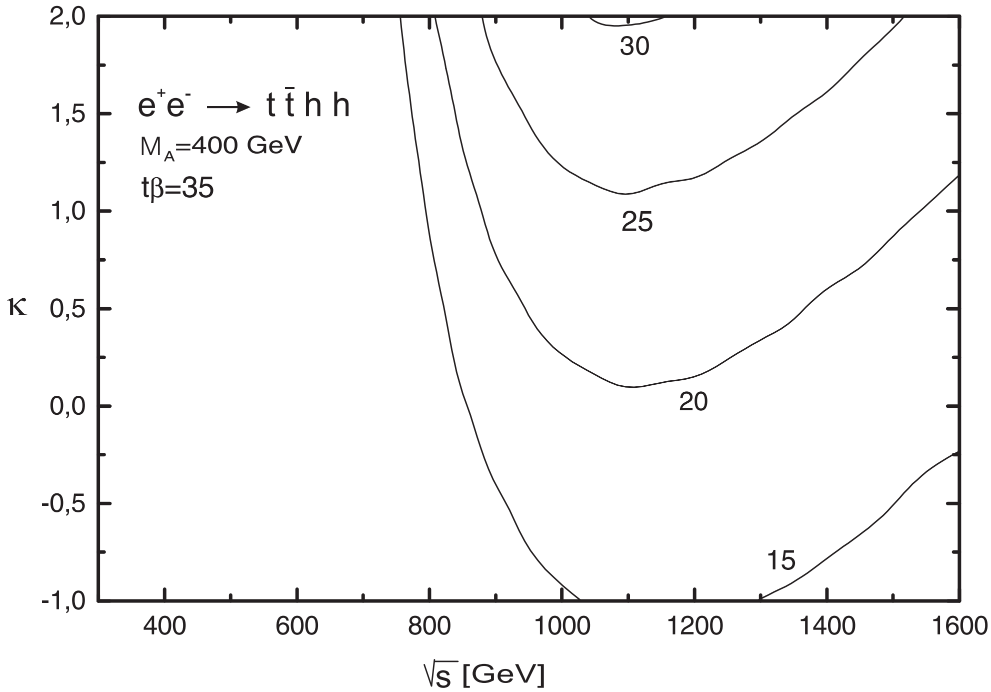

FIG. 9: The same as in Fig. 6, but for the process $e^+e^- \to t\bar{t}hh$.

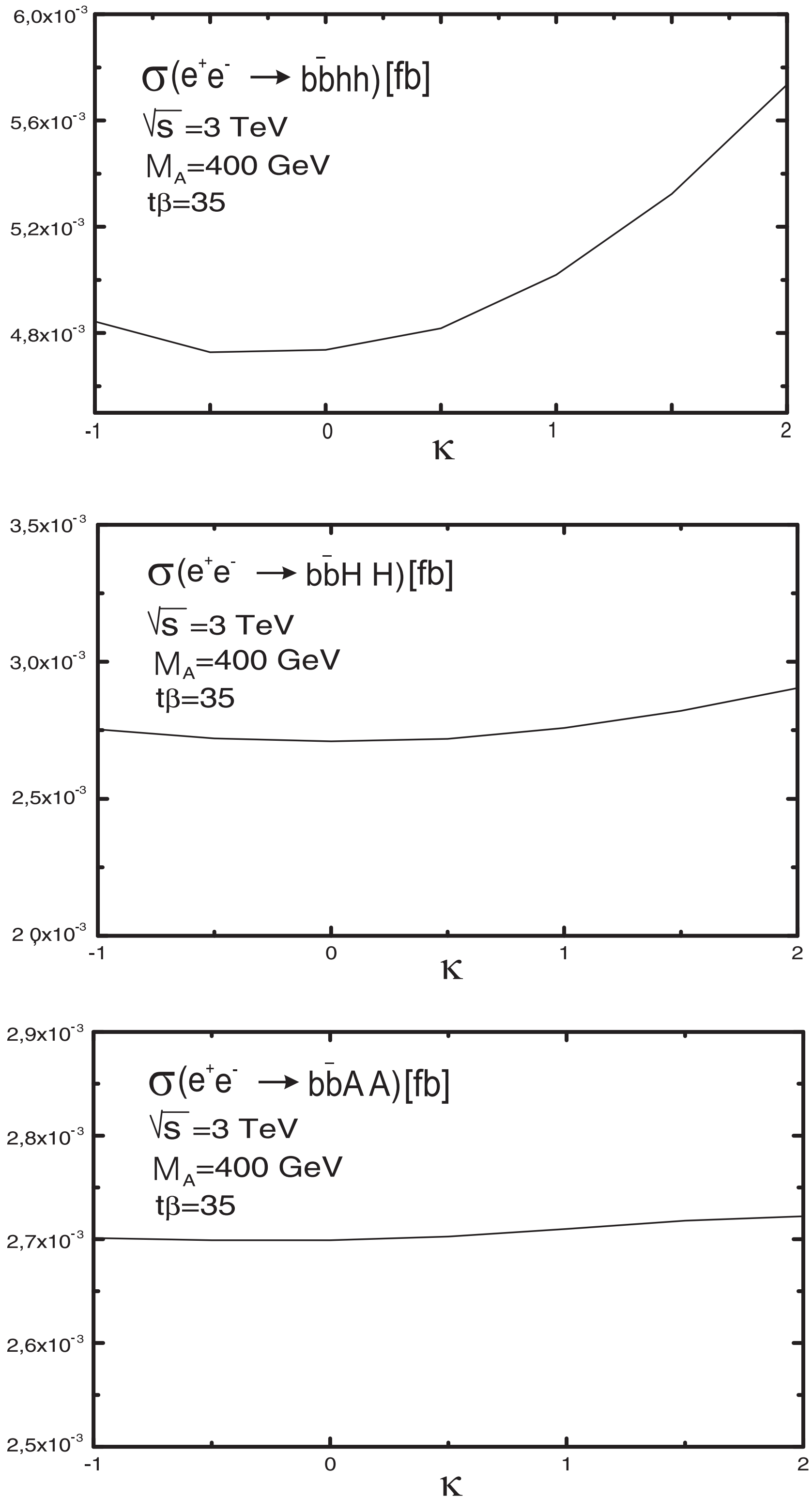

FIG. 10: The same as in Fig. 4, but for $\sqrt{s} = 3\ TeV$ with $M_A = 400\ GeV$ and $\tan\beta = 35$.

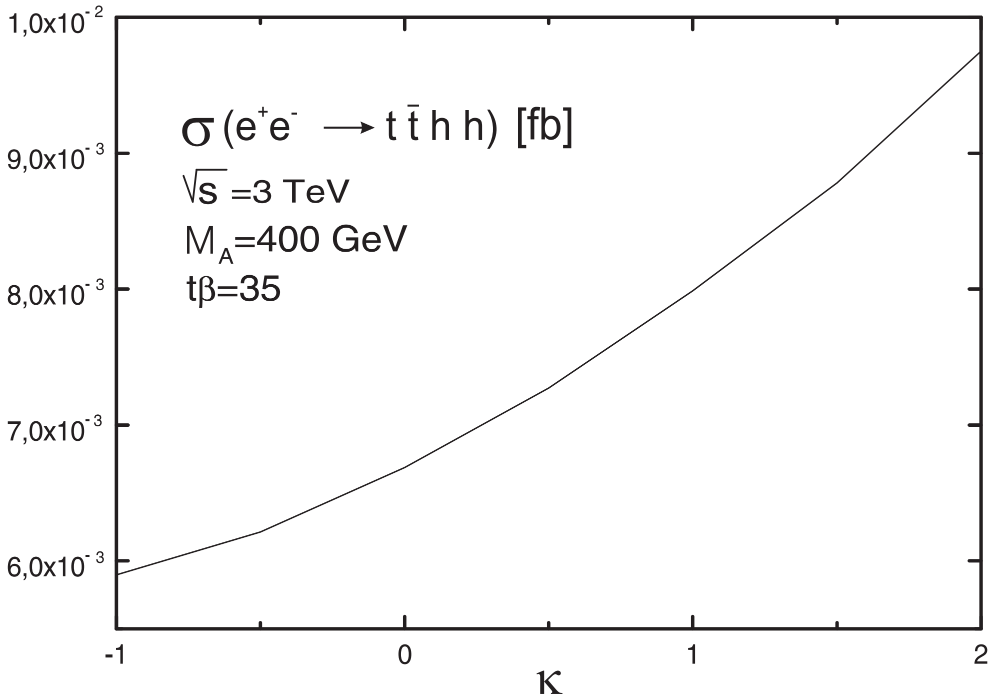

FIG. 11: The same as in Fig. 10, but for the process $e^+e^- \to t\bar{t}hh$.